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**EMPIRICAL SUPPORT FOR ASYMMETRY OF THE  
DISTRIBUTION OF EFFORT**

**by**

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# **EMPIRICAL SUPPORT FOR ASYMMETRY OF THE DISTRIBUTION OF EFFORT**

## **ABSTRACT**

When employers observe imperfect measures of worker effort, theorists typically assume that the observation of effort is unimodal and symmetrically distributed. This paper presents empirical evidence from two experimental work environments that question the assumption of symmetric distributions of observed effort. For these piece-rate work environments we find that observed effort is significantly negatively skewed (i.e., modal  $>$  mean effort). Two possible explanations are intra-period learning and/or on-the-job leisure. There are both theoretical and practical implications of this asymmetry. Some implications that are discussed, include: self-selection into rank-order tournaments, optimal wage spreads in rank-order tournaments, and optimal wage contracts with asymmetric information.



## EMPIRICAL SUPPORT FOR ASYMMETRY OF THE DISTRIBUTION OF EFFORT\*

### I. Introduction

An employee's supply of effort in the work place is of fundamental importance to the success of the firm. Even though a random element may exist in the production process, many firms monitor employees' output to ensure sufficient effort. Researchers should be concerned with how the random nature of observed effort may affect work place policies (e.g., sanctioning or rewarding, promotions, wage spreads, etc.) that are output-based. If a work place sanction, for example, is based on effort observed above or below some threshold, then an asymmetric distribution of observed effort would not generate the same penalty probabilities as a symmetric distribution. Such miscalculation of the effort distribution may also affect the employer's optimal wage structure for the employees. The end result is that theoretical assumptions about the nature of such observed effort become a critical step that can affect not only the predictions but also the work place implications of any theory.

Existing studies addressing the moral hazard that accompanies the inability to observe true effort can be found in the tournament theory and internal labor market literatures. Mathematical elegance and convenience are understandable reasons behind some standard assumptions about the nature of an error term. Different versions of random production or observed effort can be found in Lazear and Rosen (1981), Malcomson (1984), Parsons (1986), and Yun (1997), among others.<sup>1</sup> Yun notes that the assumption of a well-behaved error term is "...standard but critical." It is clear that, when using a random element to describe the uncertain

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\*This paper has benefitted from discussions with Takao Kato, Jill Tiefenthaler, Jyoti Khanna, Gerald Sohan, and participants in the 1998 Economic Science Association meetings in Tucson, Arizona.

<sup>1</sup>Lazear and Rosen assume that lifetime output is based on both effort and a well-behaved random error term, Malcomson relates output to effort with a symmetric and unimodal error term, Parsons gives several models which assume the value of a worker's output is a function of effort and a random zero-mean error-term, and Yun includes a well-behaved error term in relating an employee's true effort to the employer's observation of that effort.

link between effort and productivity, the standard practice is to formulate theories incorporating well-behaved error terms that generate symmetric distributions.

This paper presents experimental evidence that questions the validity of the assumption of a symmetric distribution of effort when certain plausible conditions are present. Controlled experimentation allows us to generate data that might not otherwise be available, such as work effort data. While surveys that provide effort data, though rare, are an alternative, aggregate responses (i.e., responses per labor period) do not allow for distributions of intra-period effort to be studied. Further, such surveys are subject to possible recall or self-reporting bias.<sup>2</sup> Laboratory methods can limit the bias in the data, and they do so at a competitive cost. Perhaps most important is that such experimental methodology can generate the exact data sought after.

Dickinson (1999) reports on a series of experimental labor markets that include a very precise measurement of production times for subjects who type paragraphs for piece-rate wages. Piece-rate wages largely eliminate the concern for monitoring and ensure that higher or lower effort is based on preferences for leisure and effort, as opposed to the risk-taking behavior associated with shirking under an hourly wage scheme. The production times generated represent observed effort in the experiment. One might consider the mean production time to proxy true effort for the day, but such data also allow us to study the intraperiod distribution of production times. Once all production times are categorized, we find that the underlying distribution of production times is *not* symmetric. Two hypotheses are offered for the asymmetry in observed effort distributions: intra-period learning and on-the-job leisure.

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<sup>2</sup> Greenberg, et al. (1981) uses hours of work as a proxy for work effort in the Negative Income Tax (NIT) experiments and concludes that there is "substantial" labor supply underreporting since the experimental payments were, in part, determined by this variable.

In what follows, we assume that observed output is observed effort to highlight the distinction between the employee's true effort and what the employer actually observes (henceforth the terms "effort" and "observed effort" are synonymous, and we will use the term "true effort" to refer to the employee's latent effort as distinguished from what is observed by the employer). We find that if workers enjoy on-the-job leisure and/or display intra-period learning of the job task, then effort distributions will be negatively skewed—modal effort will be greater than mean effort. Effort outside of these learning and leisure considerations may be more "normally" distributed, but once a worker begins to accumulate observations of low effort that are not offset by equally-sized observations of high effort at some point during the work period, then the distribution becomes skewed. It then becomes important to identify which workplace environments may be more subject to learning or leisure on-the-job since policies in these workplaces may not yield the desired outcomes.

Some of the implications of an asymmetric (negatively skewed) distribution of effort include the following. When workplace policies are based on observed effort, lower probabilities of penalties and/or higher probabilities of rewards exist under this asymmetry. In the case of rewarding workers for effort observed to be above some standard, the distributional asymmetry implies larger expenditures on "prizes" than anticipated. Another implication is that the optimal wage spread for a rank-order contest may be *decreasing* in the variance of the effort distribution (the classic result is a wage spread that is increasing in this variance). Thirdly, asymmetric effort distributions also affect the extent to which effort increases with the probability of promotion since this result relies on whether the effort distribution function is increasing or decreasing around the mean level of effort. Finally, self-selection into simple-rank-contests can be achieved with higher penalty probabilities than previously known. This is not an exhaustive list of the

implications of an asymmetric distribution effort, but it should suffice to highlight our point that theories, and workplace policies, can be sensitive to the form of the distribution.

## 2. Effort Distributions

Let us suppose that a worker's true effort during a work period (e.g., an hour, day, week, etc.) is given by the mean level of effort for that period,  $\mu_e$ . Effort here might be considered as a rate of production or amount of task to be accomplished per unit of time within the work period. Within the work period the employer observes employee output and therefore has a private observation of employee effort. Call this observation of effort  $y$  that differs from the employees' true or mean effort by an error term. The employer's observation of true employee effort may be written as  $y = \mu_e + \varepsilon$  (following Yun (1997)). This form simplifies in assuming that output equals effort on average, but the point to be made is general in that we assume  $\mu_e$  is invariant within the work period. That is, all variation of effort and output within the work period is captured by  $\varepsilon$ . Across periods, however, mean effort may change due to exogenous factors (e.g., wage changes, alternative employment opportunities).

It is then clear that observed effort is a random variable. Let  $f(y)$  be the distribution function of observed effort, a function determined by the distribution of the error term. We might then interpret the error term,  $\varepsilon$ , as the variable describing the ups and downs of effort throughout the work period. The employee supplies mean effort  $\mu_e$  for the work period, but the employer may observe effort to be above or below true effort. Malcomson (1984) likewise assumes a distribution function for observed effort given true effort that depends upon the error term—the difference between observed and true effort. As Malcomson notes, such a function

may have several interpretations, but we will interpret  $f(y)$  as the distribution of observable effort (output) given true effort.<sup>3</sup>

To some researchers, output is a combination of effort and ability. We interpret ability to be an exogenous factor brought into the work environment by the worker. As such, the shape of the distribution function  $f(y)$  is unaffected by this type of ability (rather, higher ability would map a given level of true effort to a higher level of output for a work period). We will call “learning”, as opposed to ability, that which improves the mapping of effort to output *within* the work period. As such, any intra-period learning in this framework must then be captured in the distribution of  $\varepsilon$ . The points we highlight here are: 1) ability and any other incentive mechanisms (such as an increase in wages) position or shift the entire distribution of effort, and 2) our key results are derived from the shape of the distribution, not from its location.

A most common assumption would be that the error in the employer’s observation of effort is distributed not only zero-mean and symmetrically, but that the error is also unimodal (e.g., the classical assumption of the normal distribution). Yun (1997) offers a recent example of such a theoretical setup for studying rank-order contracts under asymmetric information. In Yun, a worker is effectively penalized if effort is observed to be below some threshold level,  $q$ . Figure 1a shows the situation graphically. The area in the left tail of the effort distribution gives the probability of being penalized. One might envision a “carrot” approach to motivating workers that rewards workers whose effort is observed above some level  $Q$ . An employer basing rewards

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<sup>3</sup> Possible interpretations of true effort include: hours actually worked, work completed, quality of work, etc.

or penalties on something like Figure 1 could choose  $q$  and  $Q$  as cutoff levels of effort and be ensured an equal *a priori* probability of penalizing or rewarding workers.<sup>4</sup>

### **Learning and/or Goofing off**

The distribution of effort need not be symmetric and may be affected by common occurrences in the workplace. For example, most employees take unscheduled breaks during the day. An informal break, as distinguished from scheduled, employer-aware breaks, might involve surfing the web or taking the social route down the hallway. If such breaks are not offset by equally-sized bursts of additional effort, then such on-the-job leisure will negatively skew the distribution of effort during scheduled work periods. That is, the mode of  $f(y)$  will lie to the right of the mean. This asymmetry is functionally captured in the distribution of the error term,  $\varepsilon$ . Figure 1b graphically depicts the resulting distribution of effort.

Suppose, for example, that an employee has both standard effort and some nonstandard effort, where standard effort refers to a symmetric unimodal distribution about  $\mu_e$  and the nonstandard refers to bursts of effort and goofing off. As long as goofing off is stronger than bursts of effort, the effort distribution will be skewed. If effort can be cardinally ranked, then suppose that 5 units of the “standard” effort are observed as (9,10,10,10,11)—higher numbers referring to higher effort levels. Both the mean and the mode of this effort distribution are 10. Now suppose that the employee has two bursts of effort (12 and 13), one web-surfing session (4) and one catnap (2). By allowing for nonstandard effort, we now have a distribution that would be skewed as the mode is still 10, but the mean is now 9 due to the on-the-job leisure. So, mean effort of 9 might be due to standard effort being distributed about 9, or by standard effort being

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<sup>4</sup> Strictly speaking, this result holds if all workers are assumed for simplicity to have the same mean level of effort. The result still holds for heterogeneous employee effort if we assume that employers use some sort of handicapping in their policies (e.g., use different thresholds for different workers)

distributed about 10 with some goofing off thrown in. These numbers are clearly ad hoc, but they have been chosen to reflect what is empirically supported in the data to be presented shortly. We might also note that bursts of effort are clearly limited by an employee's ability, whereas on-the-job leisure may not be subject to upper limits.

Learning (intrapersonal) would also have the same effect on the distribution of effort. Here we define learning as an initial period of below-average effort that is *not* balanced out with later above-average effort. Take our same employee from above, for example, and add to his standard effort some initially low effort as he learns the tasks to be completed. If these units of effort are, say, (6,7,8,9) then the resulting overall distribution is skewed left with mode=10 and mean=8.9. We have done nothing more than highlight what is logical. An otherwise symmetric distribution of effort will be skewed if we start loading up observations on one tail of the distribution more than on the other. The practical result is that the employer observing effort who assumes a symmetric distribution around the modal choice of intrapersonal effort would incorrectly characterize these effort distributions.

### **3. Evidence for the Asymmetry of Observed Effort**

#### **The Data Set**

Due to the lack of existing data sets that measure effort in a meticulous fashion, there has not been the opportunity to construct an empirical distribution of effort. An ideal data set would include an unbiased measure of effort on a very fine grid (e.g., multiple measurements of effort at different times per day would be necessary). Such a data set is presented in Dickinson (1999)<sup>5</sup>, where subjects complete a task repetitively for piece-rate wages. The subjects type paragraphs,



subject to a quality control check, and receive on average \$.30 per paragraph that takes the average subject about 5 minutes to type.<sup>6</sup> Subjects also received a fixed nonwage payment for each day's work, and they participated in four experimental workdays each. Two data sets are discussed and presented. Depending on which experimental design is employed, subjects either work for a fixed 2-hour work day (the Intensity Experiment) or they are allowed to leave at their choosing (the Combined Experiment). The Intensity Experiment has as the only work variable of choice the work intensity of the subject, whereas the Combined Experiment combines a choice of both work effort and hours of work. The piece-rate wage structure of the work environments may limit the generality of the data set, but piece-rate wages eliminate the need for monitoring the workers in the experiment. This implies, for example, that the consumption of on-the-job leisure reflects the subject's desire for leisure as opposed to strategic behavior aimed at reducing the probability of being caught shirking. In addition, piece-rate (or quasi piece-rate) wages are not uncommon at all in natural work environments. Examples of at least quasi piece-rate wages are found in contract work, fruit and vegetable picking, waiting tables and other work that pays tips (since tips are the majority of the worker's pay), and a wide variety of sales jobs that pay on commission.

Observed effort is measured in both experiments quite precisely. A subject, after typing his paragraph on the computer terminal, prints the paragraph to a central printer and the printer places a time stamp on each paragraph. The experimenter then checks the paragraph for errors (up to five were allowed without penalty) and stores the time stamps as an exact measure of when the unit of production was completed. The result is a series of paragraph production times

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<sup>5</sup> The principle aim of this study is to decompose a wage change into substitution and income effects and to analyze the resulting changes in work effort and/or time spent working.



for a total of 41 subjects (26 in the Combined Experiment and 15 in the Intensity Experiment) for each of 4 days. The computerized collection of the production time data is not only precise but it is also unbiased—no self-reporting, bad memory, or recording bias exists in this data set.

Different subjects are more or less skilled at typing, but each subject's workday data can be categorized by mean production time and standard deviation of production time for that day.

This final fact allows us to categorize and aggregate the effort data across subjects to construct an empirical distribution of production times (and, hence, of observed effort). Aggregating across individuals, there are 1493 observations (i.e., production times) in the Intensity Experiment and 1344 observations in the Combined Experiment.

The data is categorized as follows: For a given subject on a given experimental work day the data is grouped into 22 categories. The bandwidth for each category is one-half standard deviation ( $\sigma$ ). Category 11,  $C_{11}$ , contains the mean,  $\mu$ , of each distribution of production times such that  $C_{11}=(\mu-0.25\sigma, \mu+0.25\sigma]$ . Higher (lower) categories contain observations continually greater than (less than) the mean with each category containing the neighboring  $0.5\sigma$  interval (e.g.,  $C_{10}=(\mu-0.75\sigma, \mu-0.25\sigma]$  and  $C_{12}=(\mu+0.25\sigma, \mu+0.75\sigma]$ ). The first category  $C_1=(\mu-5.25\sigma, \mu-4.75\sigma]$  and  $C_{21}=(\mu+4.75\sigma, \mu+5.25\sigma]$ . The final category,  $C_{22}=(\mu+5.25\sigma, \infty]$  allows for any extreme outliers in terms of higher than average paragraph production times. The resulting paragraph production time distributions for both experimental designs are plotted along with an approximated standard normal distribution in Figure 2.<sup>7</sup> Given that higher effort implies lower paragraph production times, the empirically observed effort distributions can be thought of

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<sup>6</sup> If we were to ignore the fixed nonwage payment, the average subject earned less than \$4 per hour of incentive pay. This was enough to motivate the subjects, however, as the effort response to a wage change is statistically significant.

<sup>7</sup> The standard normal is approximated by using standard probabilities for Z-scores and matching the appropriate probability with the appropriate category.

as the mirror image of the production time distributions (mirrored about the mean of the distribution).

### **Data analysis and the learning/on-the-job leisure hypothesis**

What is clear from Figure 2 is that the distributions of production times are positively-skewed—which implies that observed effort is *negatively*-skewed since output is inversely related to production times. A chi-square test ( $df=21$ ) is used to compare occurrences in each of our 22 categories for the Intensity and Combined Experiments with occurrences under the standard normal distribution.<sup>8</sup> We can reject the null hypothesis that the distribution of the Intensity and Combined Experiments are each equal to the standard normal distribution ( $p<.01$  for both tests). We also reject the null hypothesis that data from the Intensity and Combined Experiments come from the same distribution ( $p<.01$ ). The more peaked distribution of the Intensity Experiment data is most likely due to the longer average workday (120 minutes versus an average workday of 53 minutes in the Combined Experiment). It is clear, though, that the asymmetry in the production time distribution is robust with respect to the particular experimental design.<sup>9</sup>

The previous section offered two hypotheses for skewness of the effort distribution: on-the-job leisure and/or intra-period learning. A central conclusion in Dickinson (1999) is that on-the-job leisure is enjoyed in both experimental designs. Whether individual workers merely slow their effort for a period of time or actually stop to take a break, we are safe in concluding that at

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<sup>8</sup> In using the chi-square test, some of the outlier data categories are aggregated so that there are no empty categories.

<sup>9</sup> In fact, the asymmetry stands up to further disaggregation of the data. When testing the distributions of each separate workday, the distributions are statistically significantly skewed right in all cases for both experimental designs. Further, one cannot reject the null hypothesis for the chi-squared two-sample test that any two given experimental days within an experimental design come from the same distribution. The one exception is that the first work day in the Combined Experiment is statistically significantly different from the other days of that design ( $p<.01$  in all cases), but its distribution is still significantly skewed left.

least part of this asymmetry in the observed effort distribution is due to on-the-job leisure. One can highlight this fact by picking out a couple of salient, albeit atypical, subjects from the Intensity Experiment. For Subject 2 on Day 4, the mean paragraph production time is 6.6 minutes, with an outlier of 36 minutes. While such an outlier contributes to a high variance in production times for that day, that particular data point is still in the far right tail of the production time distribution. It is literally impossible for the subject to come up with a burst of effort equally far out in the distribution's left tail to offset a production time of 36 minutes. Similarly, Subject 4 in the Intensity Experiment had a mean production time of 7.17 minutes on Day 4, and an outlier of **59 minutes**.<sup>10</sup> (Both of these subjects were witnessed falling asleep on their respective keyboards, and they chose this as a trade-off to earning more piece-rate wage income).

One way in which we might address the learning hypothesis would be to divide the workdays up into an initial period and the remaining workday. If intra-day learning is present, *ceteris paribus*, the data from the first part of the workday will contain the majority of the right tail of the production time distributions. Given that average Intensity work days are approximately twice as long (120 minutes) as Combined work days (53 minutes), we separate the data as follows: For the Combined data, work days are divided up into first and second half (698 and 646 observations, respectively) and for the Intensity data, work days are divided into first quarter and final three-quarters (395 and 1098 observations, respectively). Any odd observation is included in the initial time subsample. This allows for an approximate (arbitrary) 30 minutes of warm-up time for the workdays. We use the chi-square two-sample test for differences in

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<sup>10</sup> To be certain, one could only consume on-the-job leisure and no off-the-job leisure in the Intensity Experiment since the workday was fixed at 2 hours. Nonetheless, consumption of on-the-job leisure was still present in the

distributions for the subsamples of initial time versus remaining time for both the Intensity and Combined Experiments (utilizing the full sample mean and standard deviation to categorize the data. The null hypothesis is that the two subsamples come from the same population, and the alternative is that the initial portion of the workday contains more of the high production times. For both experiments we can reject the null hypothesis in favor of the alternative of a lower observed effort portion of the overall distribution for the initial period ( $p < .01$  for both experiments). Figures 3a and 3b show the decomposition of the two workday distributions.

We have seen that, for this task, intraday learning is present and partially responsible for skewing the distribution of observed effort. This is important because, while it may be the case that monitored workers are able to mask their on-the-job leisure, learning may be more difficult to hide or eliminate. In both cases, however, the *post-learning* data subsample is skewed left (see Figures 3). This clearly shows that any “learning” cannot fully account for the skewness in the distribution.<sup>11</sup> We take this to be another manifestation of the on-the-job leisure present in the data.<sup>12</sup>

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Combined Experiment and, while outlier production times were not as large in absolute terms, the production time distributions in the Combined Experiment were just as skewed as those in the Intensity Experiment.

<sup>11</sup> The fact that the initial period subsample is still skewed may be a result of allowing for too great of a warm up time. It may also be the case that fatigue could skew the distribution of effort to the right as low effort observation may pile up towards the end of the workday. While certainly an issue in any naturally occurring work environment, the relatively short duration of these experimental workdays probably avoids any serious fatigue effects.

<sup>12</sup> In all data analysis to this point, we make the assumption that the data consist of independent observations. This may be a questionable assumption and, while not prohibiting us from analyzing the resultant distribution of production times, if violated it can point to particular patterns in intraday effort. A nonparametric one-sample runs test on each separate experimental day can help indicate randomness in the data. By counting runs of production times above and below the subject's mean production time for a work day we will observe very few runs if bunching occurs in the data and many runs if severe short term cyclical fluctuations exist. Using a chi-square test for  $\alpha = .05$ , we test the null hypothesis that the data comes from a random sample by looking for “average” numbers of runs in the data. For the Combined Experiment, for only 12 out of 104 total experimental workdays could we reject the null hypothesis. For the Intensity Experiment, we reject the null hypothesis for 15 out of the total 60 workdays. In all cases, the null hypothesis is rejected due to too few runs, suggesting some bunching in the data for these work days (a two-sample chi-square test for difference in subdistributions from nonrandom days versus random days is, however, rejected for the Intensity data ( $p = .02$ ) while we fail to reject the same test for the Combined data ( $p = .20$ )). To the extent that some workday data are not random, subjects appear to group together their bursts of effort and

Overall, we can conclude that both learning and on-the-job leisure are responsible for a significant part of the distributions' asymmetry for this data set. The on-the-job leisure is probably of most concern in the workplace since its consumption is still present even when learning ceases to be an issue for a particular task. Intra-day learning may be a more serious and avoidable concern for different types of work environments where tasks alter more substantially from day to day.<sup>13</sup> We do not claim that observed effort distributions are all destined to be skewed but, for those work environments where substantial learning is present and/or on-the-job leisure is a concern, employers (and theorists) should take this into account when establishing workplace policies based on observed effort.

#### 4. Implications

In this section we highlight several implications of the asymmetric observed effort distribution. As mentioned before, an asymmetric distribution of observed effort has implications for *any* theory that assumes a typical symmetric distribution, but by discussing at least a few implications in greater detail we hope to highlight the importance that the asymmetry may have in the workplace for both workers and employers.

By aggregating all data from both Experimental Designs, we construct an overall empirical distribution of observed effort. The positive-skewness of the production time distributions implies that effort is *negatively* skewed since higher than average effort generates a

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their on-the-job leisure (when consumed through higher production times for consecutive paragraphs as opposed to one long break).

<sup>13</sup> Learning of the job in general is a different type of learning to be considered. Subjects in these experiments may, for example, require a day to get their typing skills back to their usual level. A brief check of the data reveals that the Day 1 data do not account for a disproportionate number of days determined to generate nonrandom data (a fact which does not prohibit its inclusion into the aggregate data with the other work days). This would support the claim that new employment learning is not a factor, although to the extent that it may be a factor in some jobs, I doubt that the employer would base any substantial policy on the initial workday alone.

lower than average production time. Figure 4 plots the empirical distribution of observed effort along with the standard normal distribution.<sup>14</sup> The issue then, in terms of Figure 4, is the consequence of various employer policies that depend on the employer's observation of effort when a symmetric distribution of effort is assumed (as is standard theoretical practice) versus when an asymmetric negatively skewed distribution is assumed.<sup>15</sup>

If an employer were to base reward and/or sanction policies on effort observed above or below some threshold, then it is clear that reward and penalty probabilities would be miscalculated—see Figure 4 at thresholds  $q$  and  $Q$ . One way to avoid this problem would be the practice of employers to promote, for example, the top 5% of employees (ranked by observed effort). Similarly, grading on a curve in academics would avoid this problem relative to establishing a straight-percentage grading scale if a bell curve in final grades is desired. Such observations are not derived from any theoretical model, but they do highlight the fact that policies based on absolute effort thresholds will not generate the *ex ante* desired outcomes (e.g., number of dismissals or bonuses) if symmetric effort is mistakenly assumed. We now turn to some workplace implications derived from specific theoretical models.

Malcomson (1984) develops a theory of contracts and effort in which observed (symmetrically distributed) effort of an employee determines whether or not that employee is promoted to earn a higher wage in period two of a two-period contract. This choice of the firm's optimal contract is shown to depend upon, among other things, how effort responds to the probability of promotion (p.497). For the assumed symmetric and unimodally distributed

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<sup>14</sup> The graph of the empirical effort distribution is just the mirror image of the graph of the standardized production time distributions (which are skewed right)—mirrored about the mean production time.

<sup>15</sup> This mean of the distribution is, of course, a choice variable determined primarily by the piece-rate wage offer. For this data set, an income-compensated higher wage increased effort, on average, in the Intensity Experiment,



observed effort, Malcomson shows that when observed effort is above average effort, effort is increasing in the probability of promotion,  $P$ . His conclusion, however, is really that effort is increasing in  $P$  so long as the effort p.d.f. declines.

Figure 4 shows that when effort is asymmetrically distributed, there exists levels of effort greater than mean effort such that optimally chosen effort is declining in  $P$  (since the p.d.f. is increasing). The implication on the optimal contract that the firm would offer is obvious once one considers how the firm pays for the promotion wage. Firms would not be in equilibrium if they did not state the proper  $P$  in their employment contract. Adjustment to equilibrium requires adjustment of  $P$ , and a higher  $P$  is “made up” to the firm in the form of higher employee effort. Firms would not properly adjust  $P$  if effort were assumed to increase in  $P$  when it really is declining in  $P$ . This is possible when effort is improperly assumed to follow a symmetric distribution.

As a second example, consider Lazear and Rosen (1981)—a seminal paper on rank-order tournaments. Per period output is a function of average output (call this effort) and a random well-behaved error term,  $\varepsilon$ . Per period output can then be considered observed effort.<sup>16</sup> A rank-order tournament is shown to possess the same efficiency properties as piece-rate wages for risk-neutral workers. Wages offered in the tournament are either  $W_1$  or  $W_2$  with  $W_1 > W_2$ , so that  $W_1$  is the fixed prize going to the tournament winner. A key result is that there is a unique equilibrium spread ( $W_1 - W_2$ ) that maximizes workers’ expected utility. For a risk-neutral firm,

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whereas it decreased effort (as workers opted for longer but more leisurely work days) in the Combined Experiment (see Dickinson).

<sup>16</sup> In this paper lifetime output is assumed to depend on skill or average output (e.g., effort) as well as lifetime luck represented by a well-behaved error term. Lifetime output is considered to reduce the problem to a single period to avoid sequencing effects. Of course, we encounter no problem in assuming that lifetime effort could also be asymmetrically distributed (even with a symmetric “luck” term). Our data aggregated across working days is still asymmetrically distributed.

they go on to show that when  $\varepsilon$  is normal, the optimal prize spread varies directly with the variance of the output distribution. This result relies on the fact that the difference in workers' level of investment (e.g., true effort) is compared to the difference in the workers' error term. Normally distributed error terms guarantee that this difference  $\varepsilon_i - \varepsilon_j = \xi$  is also normally distributed. With asymmetrically distributed output, this difference in errors is certainly not guaranteed to be symmetric. It can be shown that for certain left-skewed and right-skewed distributions of  $\xi$  the optimal prize spread can vary *inversely* with the variance of output.<sup>17</sup> The result is that adjustment to equilibrium with a higher output-variance workforce may be accomplished by decreasing the prize spread if workers' effort is asymmetric—a result contrary to a key Lazear and Rosen result.

Finally, consider the issue of efficient self-selection into simple rank-order contests (SRC's) as set forth by Yun (1997). As employers set an effort threshold below which workers are penalized, there exists a given probability of being penalized, and this penalty decreases in effort. A salient result in Yun is that penalty probability functions must be convex for the existence of efficient self-selection of individuals of differing abilities into tournaments of their own type (high or low ability workers). This convexity implies that when true effort—that is,  $\mu_e$ —increases above the penalty threshold, the probability of being penalized decreases at a decreasing rate. With the assumed unimodal (at effort's mean) distribution of effort, this convexity occurs for all true effort levels above the penalty threshold. Under the empirical asymmetric effort distribution, the convexity region extends down to lower true effort levels.<sup>18</sup>

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<sup>17</sup> A right-skewed gamma distribution and a left-skewed 3-parameter lognormal distribution are examples. Proofs are available from the author upon request.

<sup>18</sup> In Yun, the different choices of effort that generate the desired penalty probability functions are true effort choices. In other words, the entire distribution of effort is shifted to higher or lower true (mean) effort choices in order to



Figure 5 shows the difference in the penalty probability functions assuming the two different effort distributions. This result implies one of two things. The first possibility is that the existence of a first-best SRC holds at lower true effort levels than previously thought. The second is that if we desire the existence of a first-best SRC at the true effort levels shown by Yun, a *higher* proportion of workers could be penalized (or, alternatively, a higher penalty threshold established) under the asymmetric effort distribution than under the symmetric one.

Theory always establishes assumptions prior to developing implications. A theory or policy that performs well is one whose results are robust with respect to the relaxation of assumptions. As shown above, the assumption of symmetry of the distribution of observed effort is often a key assumption, and to relax this assumption implies that theory and/or policy results would often change.

## 5. Concluding Remarks

Existing studies that hypothesize a distribution of worker's observable effort typically assume a symmetric distribution, such as the normal distribution. Theoretical results often hinge on this assumption as would workplace policies based upon such theories. On the other hand, if effort follows an asymmetric distribution then both theory and policy are affected. Such an asymmetry in the per period distribution of effort can result from intra-period learning and/or on-the-job leisure. Intra-period learning implies that workers require an initial period of their workday where they learn or improve upon the day's task. On-the-job leisure is manifested by periods of low effort that are not matched by equally-sized bursts of high effort.

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understand the result. Our point is that the shifting of asymmetric vs. symmetric distributions does matter in terms of the theoretical implication.

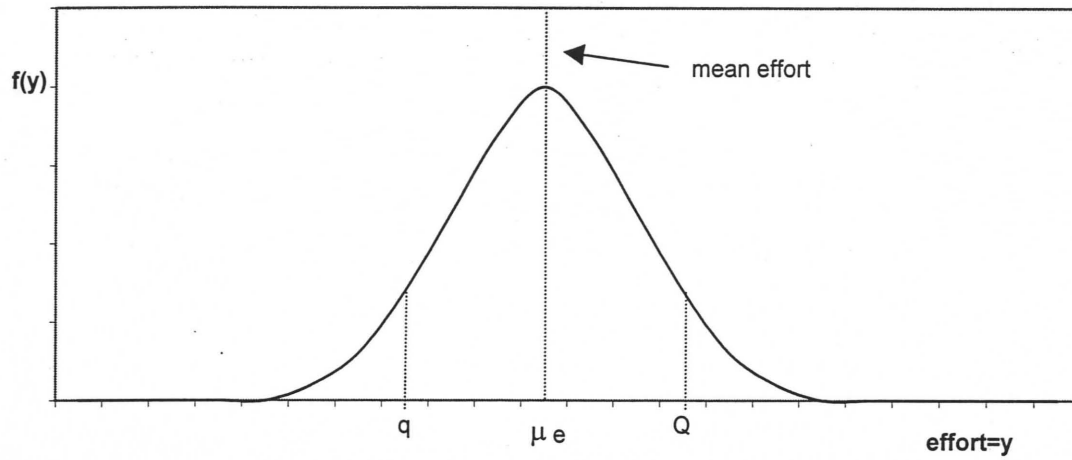
Dickinson (1999) provides a controlled set of experiments that generate data on observed work effort. Subjects type paragraphs and paragraph production times are used as a proxy for effort. All data presented point to an asymmetric distribution of effort for these work environments. We take this to imply that for those environments where intra-period learning is an issue and/or where on-the-job leisure is a concern workplace policy should be concerned about the implications of the effort asymmetry. These experimental work environments represent piece-rate wage environments, and some would argue that high or low effort is of no real consequence to the employers of piece-rate workers since low effort is rewarded with low pay. However, employers still have an interest in hard-working employees since labor inputs are often combined with other complementary inputs in the production process.

The implications of incorrectly assuming a symmetric distribution of observed effort can be found in many theories or policies based upon the assumption. These limitations range from theoretical to practical and, while not an exhaustive list, the following have been highlighted: In setting wage contracts and promotion probabilities ( $P$ ), asymmetric observed effort affects whether or not employee effort is increasing or decreasing in  $P$ —a point which is crucial to choosing the optimal wage contract. We have also seen that in rank-order contests, an asymmetric observed effort distribution can imply that the equilibrium prize spread is decreasing in the variance of observed effort. This point is contrary to an established result highlighted in Lazear and Rosen (1981). Finally, where minimum effort requirements are utilized, the existence of efficient simple-rank contests occurs with higher than previously thought penalty probabilities (and, hence, higher minimum effort requirements).

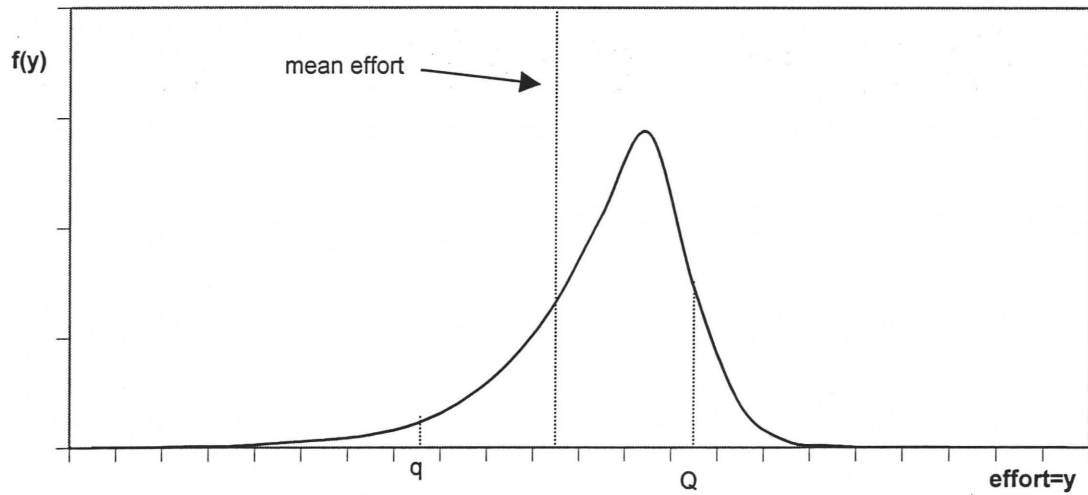
Many employers also offer positive incentives to workers surpassing some level of effort. To the extent that employers have an *a priori* idea of reward and/or penalty probabilities for their

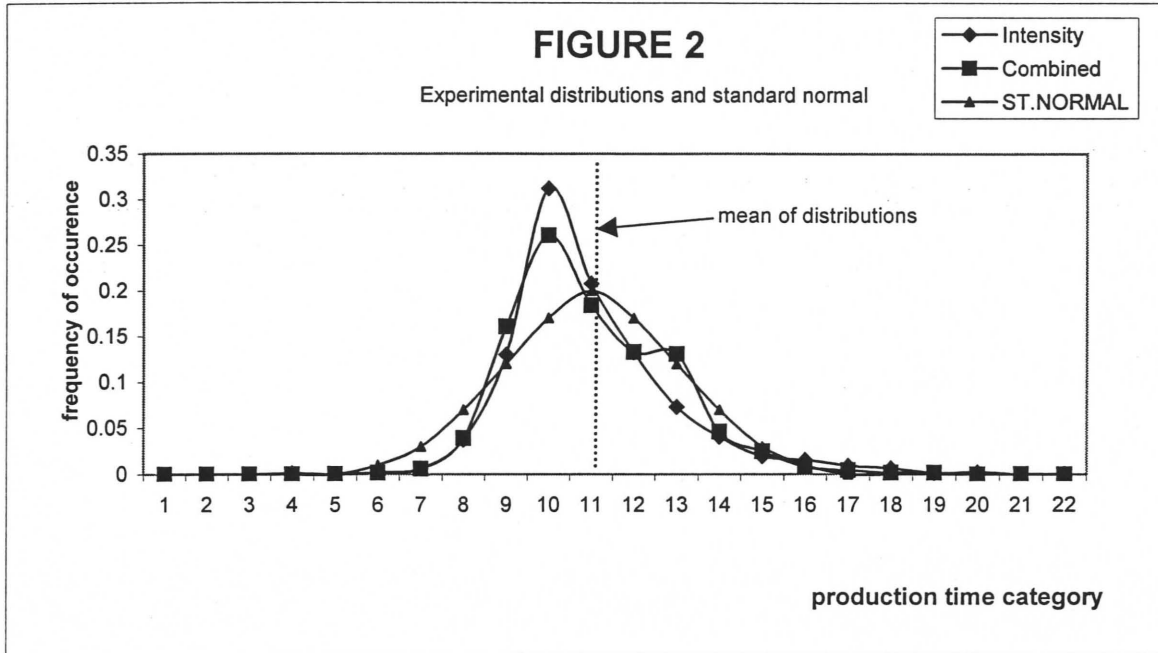
workplace policies, these probabilities will be biased in predictable ways when effort is asymmetrically distributed. Employers may penalize too few workers or reward too many or set unattainable goals for employees—all of these are, in the end, costly miscalculations. The goal of this paper has been to shed some light on the possibility of asymmetric observed effort distributions. These can occur under very reasonable workplace assumptions, and they have been documented in the laboratory work environments discussed in this paper. Future research should perhaps examine a wider variety of experimental work environments as well as attempt to gather richer field data sets that could provide high quality effort data in an attempt to establish the generality of asymmetric observed effort distributions.

**FIGURE 1a**  
Normally distributed effort



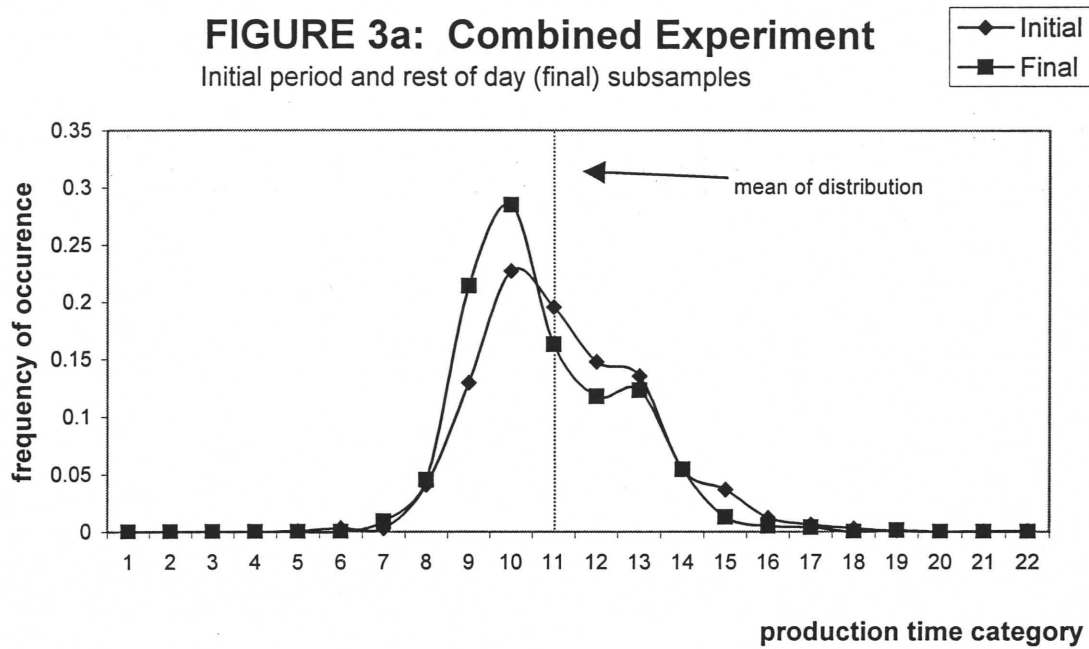
**FIGURE 1b**  
Left-Skewed distribution of effort



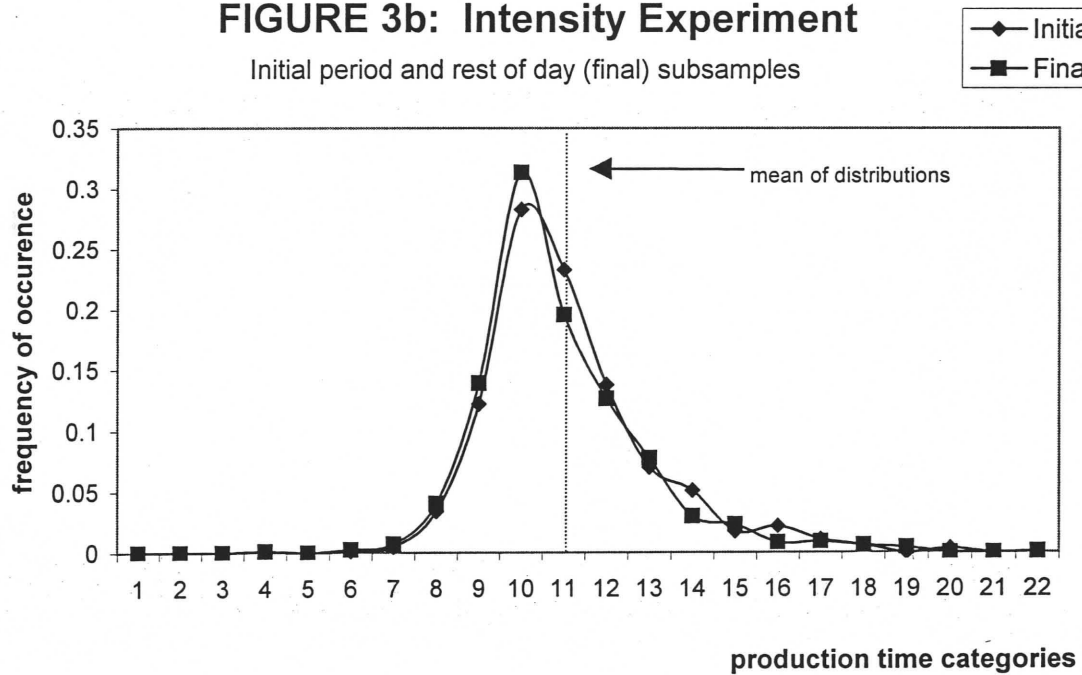


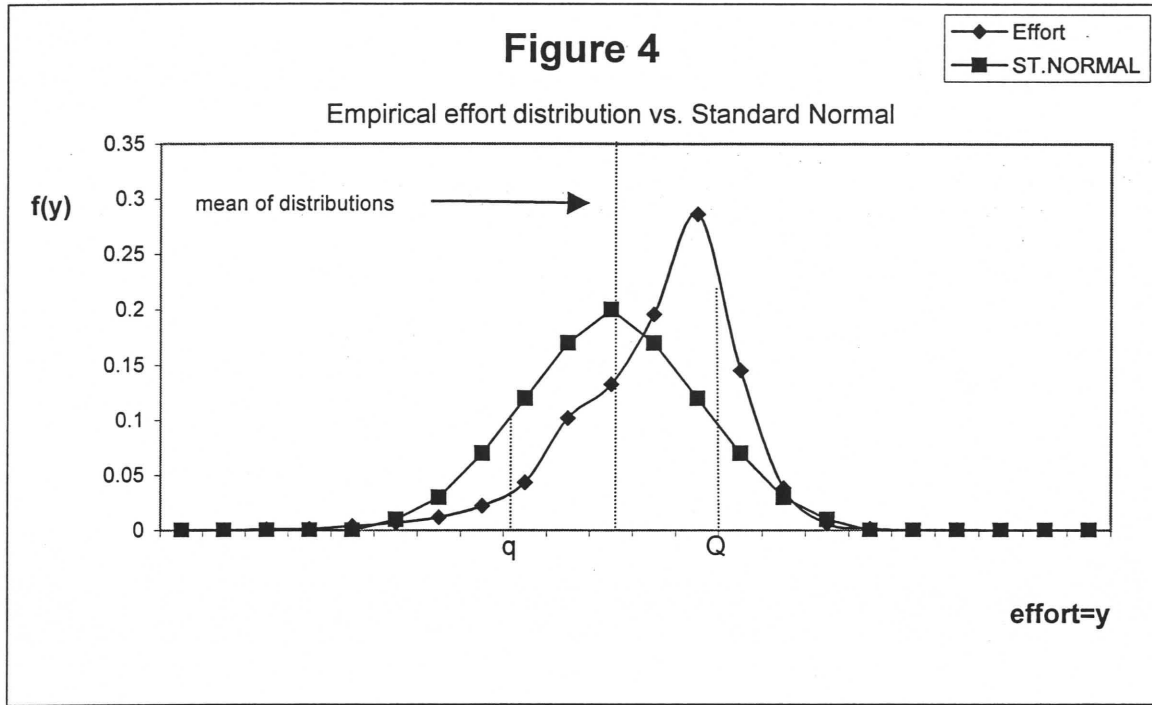
**FIGURE 3a: Combined Experiment**

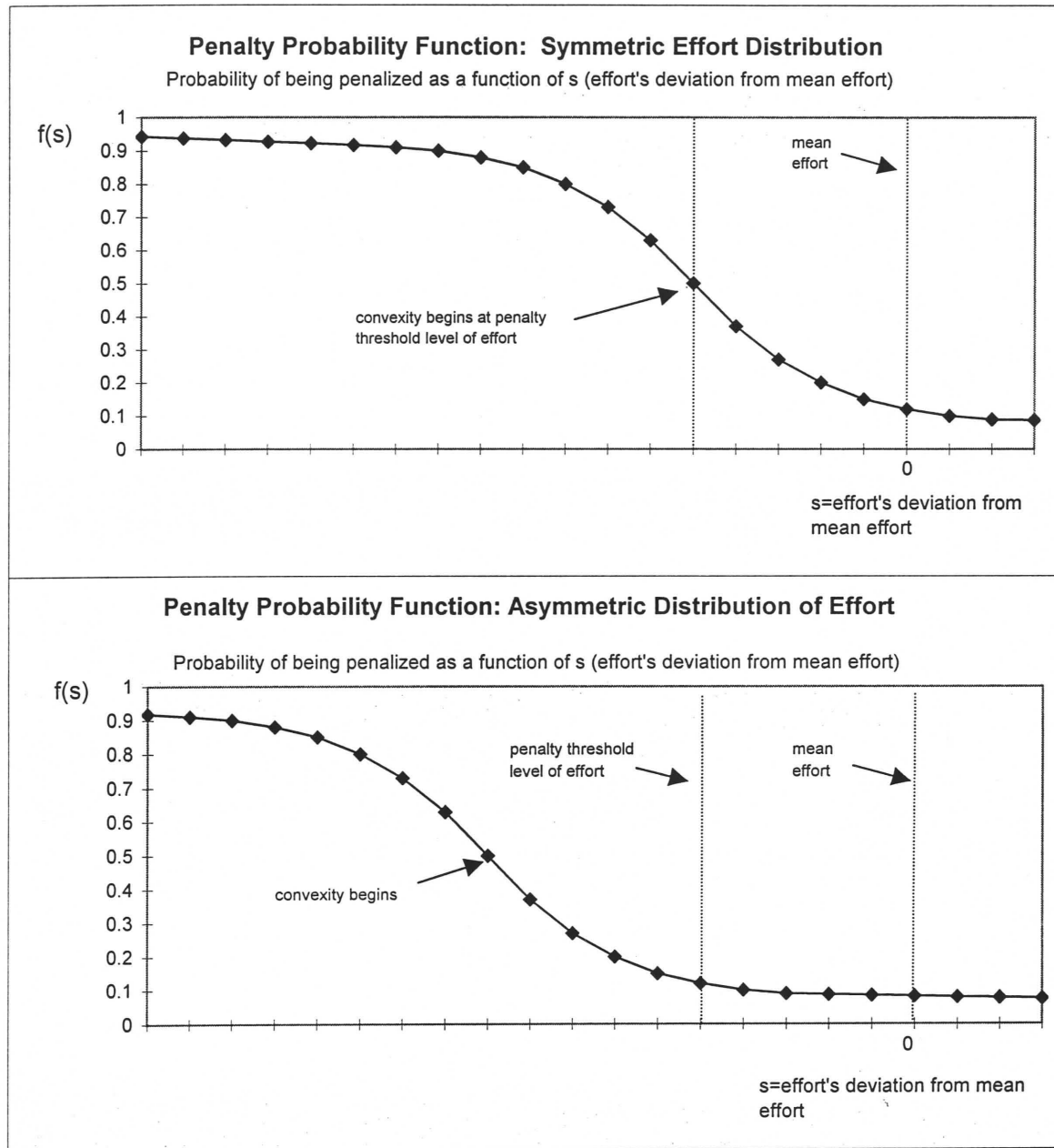
Initial period and rest of day (final) subsamples

**FIGURE 3b: Intensity Experiment**

Initial period and rest of day (final) subsamples





**FIGURE 5**



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## APPENDIX A

### Optimal prize spreads and a 3-parameter lognormal distribution of effort

A three-parameter lognormal distribution is negatively skewed. Here, we take a random variable  $x$  such that  $\theta - x$  is distributed lognormal with mean  $\mu$  and variance  $\sigma^2$ , where  $\theta$  is the upper bound of the distribution (see Aitchison and Brown, 1957). A key result in Lazear and Rosen (that the optimal prize spread is increasing in the variance of effort (their term is worker investment)) requires that the distribution of a combined error term is *decreasing* in the variance, and the result involves evaluating the p.d.f.  $f(x)$  at the mean level of  $x$  (see page 846).

This combined term's distribution follows from the distribution of the "lifetime" luck error components which combine with investment (effort) to produce output for each individual. Of course, we do not interpret the error term in this paper as luck, and we are interested in shorter work periods than the worker's entire life. Nonetheless, the tournament theory results have interesting applications to tournaments of shorter duration than an entire lifetime. Let us assume that  $w = \theta - x$ , and  $x$  is our effort variable (we can interpret  $\theta$  as maximum effort). Now since effort is left-skewed, we say that  $w$  is distributed lognormal  $(\mu, \sigma^2)$ . Since  $w$  is lognormal, we can write

its p.d.f. as  $g(w) = \frac{1}{\sigma\sqrt{2\pi}} \frac{1}{w} e^{\frac{-(\ln w - \mu)^2}{2\sigma^2}}$ . Keep in mind that  $\sigma^2$  is the variance of both  $x$  and  $w$ ,

since the distributions are mirror images of each other. As such, we need to show that we can have  $g(w=\mu)$  *increasing* in  $\sigma^2$  (Lazear and Rosen show their result holds for  $f(x=\mu=0)$  decreasing in  $\sigma^2$ ).

Evaluate the p.d.f.  $g(w)$  at  $w=\mu$  to get  $g(w=\mu)=\frac{1}{\sigma\mu\sqrt{2\pi}}e^{-\frac{(\ln\mu-\mu)^2}{2\sigma^2}}$ . Now

$$\frac{\partial g}{\partial \sigma} = \frac{-1}{\sigma^2\mu\sqrt{2\pi}}e^{-\frac{(\ln\mu-\mu)^2}{2\sigma^2}} + \frac{-1}{\sigma\mu\sqrt{2\pi}}\left(e^{-\frac{(\ln\mu-\mu)^2}{2\sigma^2}} \cdot \frac{(\ln\mu-\mu)^2}{\sigma^3}\right)$$

$$= e^{-\frac{(\ln\mu-\mu)^2}{2\sigma^2}} \left[ \frac{(\ln\mu-\mu)^2}{\sigma^4\mu\sqrt{2\pi}} - \frac{1}{\sigma^2\mu\sqrt{2\pi}} \right] \text{ which is } > 0 \text{ if } \frac{(\ln\mu-\mu)^2}{\sigma^4\mu\sqrt{2\pi}} > \frac{1}{\sigma^2\mu\sqrt{2\pi}}$$

$$\text{which implies } \frac{(\ln\mu-\mu)^2}{\sigma^2} > \frac{1}{\sigma^2\mu\sqrt{2\pi}}.$$

This implies that  $\frac{\partial g}{\partial \sigma} > 0$  if  $(\ln\mu-\mu)^2 > \sigma^2$ .

While the above condition may not hold for high variance scenarios, what we have shown is that conditions exist for which a key result from Lazear and Rosen does not hold—this occurs when we relax the assumption of a symmetric distribution of the error term.

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**For Referees:** In reference to footnote 17 in the manuscript

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## APPENDIX B

### Optimal prize spreads and a Gamma distribution of effort

The p.d.f. of the gamma distribution is given by  $f(x) = \frac{1}{\beta^\alpha \Gamma(\alpha)} x^{\alpha-1} e^{-x/\beta}$  for  $x, \alpha, \beta > 0$ . The

mean of the distribution is  $\alpha\beta$  and the variance is  $\alpha\beta^2$ . As such, the variance can increase as a result of a rise in either  $\alpha$  or  $\beta$ . A key result in Lazear and Rosen (that the optimal prize spread is increasing in the variance of effort (their term is worker investment)) requires that the distribution function of a combined error term is *decreasing* in the variance, and the result involves evaluating the p.d.f.  $f(x)$  at the mean level of  $x$  (see page 846). As such, we have

$f(x=\alpha\beta) = \frac{1}{\beta^\alpha \Gamma(\alpha)} \alpha \beta^{\alpha-1} e^{-\alpha}$  for the gamma distribution. By evaluating how this function changes

with respect to a change in  $\beta$ , we have

$$\frac{\partial f}{\partial \beta} = \frac{-\alpha \beta^{-\alpha-1}}{\Gamma(\alpha)} \alpha \beta^{\alpha-1} e^{-\alpha} + \frac{e^{-\alpha}}{\beta^\alpha \Gamma(\alpha)} (\alpha^2 - \alpha) \beta^{\alpha-2}$$

Given that  $\Gamma(\alpha), \alpha, \beta > 0$ , the first term in  $\frac{\partial f}{\partial \beta}$  will be negative and the second term positive for

$\alpha > 1$ . Together this implies that  $\frac{\partial f}{\partial \beta} > 0$  is possible, which would imply that the optimal prize

spread could be a decreasing function of the variance of output. Of course, there are other

distribution functions which can produce a skewed distribution of output for which the optimal

prize spread is still increasing in the variance of the distribution. Even for the gamma

distribution used above, it is possible that the optimal prize spread is still increasing in the

variance of the distribution depending upon the values of  $\alpha$  and  $\beta$ . What we point out above is

that the result may *not* hold, depending on the distribution function and parameters used.